How to model power prices: What we’ve learnt from the fast pace evolution of the German/Central European market

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What is the talk about?

E.ON founded around the liberalization of the German (and European) energy market (around the year 2000) \(\leftrightarrow\) Coincidences roughly with the beginning of **Energy Finance = Quantitative Finance / Analysis of Energy Markets**

- Not exactly: Started out in the 1990's with the US energy markets

Energy finance started out by taking concepts from general quantitative finance. However, people soon learned to observe specifics of the energy markets. Eg, need to take care of risk by highly spiking electricity prices (example of the California market).

This talk about what we have been learning about energy finance along the **evolution of the European market** (specifically: electricity). Within this relatively short period of time a lot of new market features have evolved (some surprises, too)

\(\rightarrow\) „Meta-learning“: **Expect the change**. Producing, trading, and consuming electricity tomorrow will be different from today. (Better not assume it goes the way it always has been, like, eg, trading gold)
The evolution of German power (spot) prices

- Introduction emission certificates
- Commodities surge
- Emission price breaks down
- Negative prices permitted
- Economic crisis = load demand slump
- Rise of renewables

Hourly spot price
Coal price
Wind power capacity
Photovoltaics capacity
Power price (spot) models: The basic 2-factor model – „old“ but still a useful standard

What about simple geometric Brownian motion for the spot price $S(t)$ (enhanced by seasonality)?

→ Not recommended, too simplistic.


→ Insight: The long-term dynamics $Y(t) \sim$ forward (products) shows GBM like „free“ diffusion. The short-term dynamics $X(t) \sim$ spot mean-reverts to the long-term / forward level

\[
\log S(t) = d(t) + X(t) + Y(T) \\
\begin{align*}
  dX(t) &= -\kappa X(t) \, dt + \sigma_X \, dW_X(t) \\
  dY(t) &= \sigma_Y \, dW_Y(t)
\end{align*}
\]

Seasonal deterministic component $d(t)$
Power price (spot) models: The basic 2-factor model – „old“ but still a useful standard

Results:

- Positive
  - Yields explicit formulas: forward curve (= expected price), price distributions, options.
  - Allows for calibration to observed market forward curve and spot volatility.

- Shortcomings
  - Log-normal world, no fat tailed distributions. No spikes possible
  - No negative prices possible
Spot prices often have spikes

When high demand, eg winter, hits difficult supply situation, eg high gas prices / plant outage, prices can surge up

→ Insight: Prices don’t move, they jump. And jump back to normal as soon as situation eases. This is because electricity „inelastic“: demand and supply have to match immediately and tightly

A regime switch model is suitable: De Jong (2006). Be x(t) = Log(daily spot price)

• A Markov process holds the probabilities that the price process jumps between regimes

\[
dx^M_i = \alpha(\mu - x^M_{i-1}) + \sigma \cdot \epsilon_i
\]

\[
x^S_i = \mu + \sum_{i=1}^{n+1} Z_{t,i}
\]

High spike regime H:

\[Z_{t,i} \sim N(\mu^H, \sigma^H), \quad n_i \sim POI(\lambda^H), \quad \mu^H > 0\]

Low spike regime L:

\[Z_{t,i} \sim N(\mu^L, \sigma^L), \quad n_i \sim POI(\lambda^L), \quad \mu^L < 0\]

Markov transition matrix:

\[
\Pi = \begin{bmatrix}
1 - \pi^{ML} & -\pi^{ML} & \pi^{MH} & \pi^{ML} \\
\pi^{HM} & 1 - \pi^{HM} & 0 & 0 \\
\pi^{LM} & 0 & 1 - \pi^{LM}
\end{bmatrix}
\]
Negative prices and negative spikes

Schneider (2011):

What to do about negative prices? The usual finance log transformation $p \rightarrow \ln(p)$ does not work

Observations:

- Prices are often around 0, above, as well as below
- Extraordinarily negative prices appear like inverted spikes (the „traditional“ upwards jumping spike)

Conclusion: in contrast to, eg, stock prices, the electricity price is not „repelled“ by 0. In fact, the $\approx 0$ region appears to belong to a normal price regime. Spikes can jump in both directions

Proposal (Schneider 2011): area hyperbolic sine transformation instead of log ($\zeta : offset$, $\lambda : scale$)

- Natural extension of log - Same asymptotics as $(-)\ln[(-)p]$ for large $|p|$  
- Deeper idea: Reflects the merit order feature of electricity production

→ It is still possible – similar to the log price case – to obtain closed form solutions for price distributions, forwards and European options

\[ x = \sinh^{-1}\left(\frac{p - \zeta}{\lambda}\right) \]
Co-integration

Following the works of De Jong & Schneider (2009) and Döttling & Heider (2013) here:

Rationale:

- Pairs of commodities are often linked together by production relationships. Eg: Buy fuel and produce electricity by burning it. So, price of electricity $P$ co-moves with price of fuel $F$: $P \approx \frac{1}{{\text{eff}}} F$, with eff the efficiency of power plant.

- Correlation concept not sufficient because instantaneous in time only.

→ Link the stochastic price processes by co-integration

Co-integration is not a new concept, but it requires some thinking how to sensibly employ it for energy prices. Exemplary model system:

$$dF(t) = m_F F(t) \, dt + \sigma_F F(t) \, dW_F$$

$$dP(t) = \kappa (c + b \log F(t) - \log P(t) ) \, P(t) \, dt + \sigma_P P(t) \, dW_P$$

- The fuel $F$ is the „driving commoditiy“ (and follows a standard GBM process)

- $P$ mean-reverts to a fundamental production relationship with $F$

- Note: $F$ and $P$ are additionally also correlated, eg, by weather „shocks“.
Co-integration

c + b \log F(t) – \log P(t) = 0 defines the (log) spread between the commodities.

Eg, the **German dark spread** denotes the relation between the German power price and the hard coal price. Hard coal (power plants) is the typical price setting fuel in Germany.

- Forward Example: API#2 coal in €/ton (driving commodity) and German power €/MWh year ahead 1/1/2010 till 31/12/2011.
- Spot example: see introductory slides.
Co-integration

Be \( F(t) \) and \( P(t) \) are prices of forward products, study their stochastic co-movement between initial point in time \( s \) and \( t > s \).

- We then observe for time to maturity \( T=t-s \) that the final state (terminal variance and covariance) is equivalent to assuming a **term structure of correlation**: starting with the instantaneous correlation, approaching 1 with \( T \to \infty \).
- We can use this insight to price a **spread option** (on the pair of forward products).

\( \rightarrow \) This is a useful result to value a (simplified) power plant: The future operation of a power plant constitutes a **real option**. Can hedge the option with today's fuel and power forwards in the market.
Co-integration

Some **shortcomings** though:

- Real power plant is more complicated than a linear relation between fuel and production costs: Constraints, efficiency (strongly) dependent on power output level, etc.

- The log spread is only and approximation to real world absolute price spread.

- Most importantly: The constant offset c in the definition of spread is not so constant in reality. The, eg, German power price depends on a mix of productions (coal, gas, etc – but can model a mix, too). The mix, though, varies with time: Availabilities of plant types in the short run, plants entering and leaving the system in the long run, regulations, …
Structural / Hybrid models

Following the works of Wagner (2012) and Gifvars (2013) here:

Rationale:

• History proved that „structures“ of energy market has been changing frequently and rapidly (eg, rapid introduction of large photovoltaics capacity in Germany).
  • „Classical“ stochastic modelling, though, relies heavily on market history for parameter estimation
  • Electricity prices (obviously) strongly depend on more basic quantities: load demand, fuel prices, generation assets (see also cointegration section)

→ Do not stochastically model the power price directly, but treat it as a function of stochastic processes of the basic quantities (eg, load). The function contains the merit order of the market (plants and their costs producing the load).

• Promising advantage: it is more natural and easier to have a forward/future view on the basic quantities (eg, load demand, plant park) than on the power price itself.

Aim: Combine advantages of „classical stochastic modelling“ and „fundamental / economical (equilibrium) modelling“.
Structural / Hybrid models

Stochastic processes for loads

- **Load consumption** $L(t)$ (domestic consumption)
- **Grid infeed renewables**, eg wind $L_W(t)$ and photovoltaics $L_P(t)$

Subsume to „residual load“ $R(t) = L(t) - L_W(t) - L_P(t)$. Renewables are fed into the grid with priority, so, the residual load $R(T)$ is the remaining part of consumption to be covered with conventional production (eg, coal plants). Their merit order cuve leads to the power price = **marginal plant**.

Load processes can be more easily modelled by time series approaches than the price. Eg, of the type

**Ornstein-Uhlenbeck** (OU) process plus seasonality:

$$L(t) = \psi(t) + l(t), \quad dL(t) = -\theta \, l(t) \, dt + \sigma \, dW(t)$$

- $\psi(t)$ : seasonal deterministic. Can introduce a **view of future demand/load** here.
- $l(t)$ : (mean-reverting) OU process for load residuals

...
Structural / Hybrid models

… Same type of stochastic process useful for renewables load infeed. Key point to introduce a forward view. Do not model infeed \( l(t) \) directly but "efficiency" \( E(T) = \frac{l(t)}{C(t)} \) infeed relative to installed capacity \( C(t) \). \( E(t) \) exhibits the stationary behavior we require for time series modeling.

→ Furthermore: We have reasonable predictions for future \( C(t) \)!

Merit order:
It can be more practical to use a "supply function" \( f \) instead of the fundamental order stack.

• Price for electricity can significantly differ from marginal price to cover load \( L(t) \).

→ Fit an empirical curve \( f \) to the observed relations between residual load and price.

→ Find spot price as \( S(t) = f(R(t)) \)

(a) Wagner model applied on the peak hour
(b) Wagner model applied on the off peak hour modeling data
Structural / Hybrid models

Results:

• + Model has explanatory power!

• - / Shortcoming: Spot prices have yet more structure and variability (see „point cloud“ around supply function). In the auction
  • Offer price for plant can significantly differ from pure fuel price based production costs: start cost recovery, fixed costs, etc.
  • Consumer bid price variable, too (depending on demand-supply balance)

Perspective: where to go in power price modelling?

Questions to struggle with (some examples):

- Can deal with daily power prices, but fully synthetic hourly „price profiles“ (over a day) a problem.
  - Eg, co-integration between hours through „production modes“
  - Pragmatic solution: „guided“ sampling from history
- Interconnection of markets. Eg, EPEX runs a big complex model to calculate the optimal prices and flows every day.

→ Modelling strategy? Potentially no „one model fits all“
- Choice of model dependent on valuation task at hand
Perspective: where to go in power price modelling?

Important: **model risk**!

- With the financial crisis it became clear to a wider audience that models and parameters come with their own risk
- Energy case: Models come with a lot of parameters, many of them badly determined because not or only thinly traded.

Recall the „meta learning“ – „expect the change“:

- Renewables increased the volatility of the system (Germany). It was recently decided that the spot market hourly products no longer matches supply and demand well → ¼ hourly spot prices to come as the new underlying → „Price profile“ of a day will look quite different all of a sudden
- Future of the system?
  - Share of renewables?
  - How much storage? Eg, Connection to Scandinavian hydro reservoirs, batteries, ..
  - Capacity market? (parts of the plant park provide balancing backup for the grid)

Important to keep in mind: Providing energy is at the heart of an economy → **Frequent re-adjustment of rules, supply and demand modalities**.
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Appendix

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Electricity price basics: Spot auction

The (physical) **spot market** – example **EPEX** (Germany, Netherlands, …) for a typical Central European spot market (differing designs exist for, e.g., North America)

- **Day-ahead auction**: participants submit bids and offers with respective volumes, auctioneer matches bid (supply) and offer (demand) stacks.

- The stack reflects a large span of production modes and costs. As well as load consumers
  - Base load production (nuclear, coal), peak load production (gas), almost costless renewables
  - Stack contains the “plants merit order curve”. Price setting plant = “plant at the margin”

- Non-storability & inelastic demand & technical constraints cause surprising (even extreme) price variability
  - (positive) Spikes: Generation is at the limit, very expensive production + scarcity premium
  - Negative prices: generation oversupply (e.g., extreme wind) & technical constraints